Stressing of Migration Matrices for IFRS 9 and ICAAP Calculations

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Stressing of Migration Matrices for IFRS 9 and ICAAP Calculations

Jiří Witzany[1]

Abstract:
Rating transition matrices have become a workhorse of the IFRS 9 expected credit loss and ICAAP stress test modelling. The standard method to stress a through-the-cycle transition matrix is based on a single factor Gaussian model with a correlation parameter that is usually estimated on the level of a product pool. The goal of the paper is to generalize the model allowing for more general distributional assumptions and to test empirically the sensitivity of the results with respect to these assumptions and different possible approaches to the correlation parameter estimation. We are not aware of any such empirical study in the literature. The results show that this dependence is very strong with the standard approach underestimating the results, as we argue, of a more precise calculation many times. Therefore, there is a significant model risk that needs to be taken into account in ICAAP/IFRS 9 implementation and dealt with in further research.

Key words: ICAAP, IFRS 9 provisions, correlation, Probit model, Logit model

JEL classification: G20, G28, C51

1 Introduction

The IFRS 9 new accounting standards (IASB, 2014) that have become effective in 2018 require banks to calculate and account provisions based on the principle of expected credit losses (ECL). The ECL should be forward-looking and point-in-time (PIT). It means that banks should use all available actual information on an exposure and obligor as well as the information on the current and expected (forecasted) macroeconomic development. All exposures should be classified to three stages where the third one contains the non-performing (i.e. defaulted exposures), the second one the exposures with significant increase in credit risk compared to the risk at origination, and the first one the remaining exposures, i.e. low risk exposures without significant deterioration. The ECL for Stage 1 exposures should be estimated in 12-month horizon, while for Stage 2 and 3 exposures in the life-time horizon, i.e. until maturity of each respective exposure. It means that banks need to condition their expected credit losses on macroeconomic forecasts that go significantly beyond the one-year horizon. In addition, IFRS 9 requires banks to consider only the expected (baseline) macroeconomic scenario, but also at least two additional scenario, one pessimistic (adverse) and one optimistic. The final ECL

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should be then calculated as a probability weighted average of the ECLs estimated conditional on the various macroeconomic scenarios.

A similar modelling challenge follows from the ICAAP (Internal Capital Assessment Process) stress testing programs required by the Basel regulatory documents (BCBS 2006, 2018, ECB 2018, or EBA 2018) or the regulatory stress testing programs such as the EU wide stress test (EBA 2019). The banks firstly need to define appropriate economic stress scenario, and then conditionally estimate their future financial performance, capital adequacy and other key indicators. Again, the expected credit losses will usually play the most important role in this exercise.

The ECL calculation methodology is typically based on a decomposition of the expected losses to the probability of default (PD), loss given default (LGD), and exposure at default (EAD) components where PDs driven by internal ratings play the key role. Possible approaches to estimate not only the one-year but also lifetime PD include the technique of survival analysis or the rating transition matrices modelling (see Witzany 2017 or Miu and Ozdemir, 2017). The latter approach might be considered more practical since it allows estimating not only the lifetime PDs but also probabilities of transitions between IFRS 9 stages that are need in the stress testing exercises. Therefore, we are going to focus on the transition matrices approach. Practically, a bank firstly estimates a through-the-cycle (TTC) matrix based on the historical data recording migrations between the internal rating grades (including the state of default), and then the goal is to adjust the TTC matrix conditional on a macroeconomic scenario. The adjustments need to be made for a number of future periods obtaining a sequence of conditional transition matrices (e.g. on annual or quarterly basis). In order to obtain PDs over longer time horizons, the matrices are multiplied through implicitly assuming that the rating transition process satisfies the Markov chain property.

The most difficult element of the transition matrix approach is their stressing conditional on a given scenario. The standard approach is based on a single-factor Gaussian (Vasicek's) model (see Belkin 1998 or Yang 2015) where the event of default is driven by a latent standard normal variable decomposed into systematic and idiosyncratic components. The past systematic factors and their loading (default correlation) can be derived from historical default rate series, and then predicted using a macroeconomic model. The forward-looking systematic factors can be then used to stress the product level PDs as well as the transition probabilities in the context of the model. There are two weaknesses of the model to discuss. First, the model implicitly assumes that the sensitivity (loading factor) of the transition probabilities is the same independently on the initial and target ratings. Secondly, the assumption of normality when modelling defaults turns out to be problematic, in particular in crisis periods (see e.g. Witzany, 2013). These problems have been partially dealt with in Nickell et al. (2000) implementing the ordered Probit model for different initial (Moody's) ratings (see also Miu and Ozdemir, 2008), or in Malik and Thomas (2012) applying ordered logistic regression for different initial rating on a consumer loan dataset. Alternative modelling and distributional assumptions can be found

in Figlewski et al. (2012) applying Cox regression with multiple factors to estimate separately migrations between Investment, Non-investment, and the Default state, or in Perederiy (2017) proposing a sparse Structural Student distribution-based Approach for Rating Transitions.

The main goal of our study as outlined above is to compare several approaches to stressing of transitions matrices using the Moody's corporate rating history and default data database (1920-2011). Specifically, we consider the basic Gaussian single-factor model and compare it to the models where the systematic factor loading (default) correlation depends on the initial (or even target rating) and where the Gaussian distribution is replaced by the logistic one. The following section will outline the details of our methodology. The dataset and the empirical results will be reported in Section 3 and summarized in the concluding Section 4.

2 The Single-Factor Default Model

The key assumption of the single-factor Gaussian model (Vasicek) is that the event of default of an obligor i is driven by a standard normal variable

$$Y_i = \sqrt{\rho}Z + \sqrt{1-\rho}\xi_i, \tag{1}$$

where $Z \sim N(0,1)$ is the systematic factor and $\xi_i \sim N(0,1)$ is an i.i.d. obligor specific factor. The variable Y_i can be either interpreted as the quantile-to-quantile transformed time-to-default variable or as the standardized asset return (see e.g. Vasicek 1987 or Witzany 2017). Hence, the parameter ρ representing the correlation between the default factors of different obligors can be called the asset return correlation. The unconditional probability of default of the obligor over a given time horizon (e.g. one year) can be in this context expressed as $PD_i = \Pr[Y_i \leq b_i] = \Phi(b_i)$ where b_i is a default threshold. Therefore, if the probability of default PD_i is given, then we have $b_i = \Phi^{-1}(PD_i)$. This model allows to condition the probability of default on the systematic factor value Z and obtain the well-known Vasicek's formula used by Basel II regulation (BCBS, 2006):

$$\begin{aligned} PD_i(Z) &= \Pr\left[\sqrt{\rho}Z + \sqrt{1-\rho}\xi_i Y_i \leq \Phi^{-1}(PD_i)|Z\right] = \\ &= \Pr\left[\xi_i \leq \frac{\Phi^{-1}(PD_i) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right] = \Phi\left(\frac{\Phi^{-1}(PD_i) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right). \end{aligned} \tag{2}$$

The conditional probability of default can be used as a proxy of the future default rate driven by the unknown systematic factor on a large homogenous portfolio, i.e. on a portfolio with many exposures with the same unconditional PD and the same correlation parameter ρ. In practice, this may apply to a retail product portfolio of exposures with the same rating grade, and so with approximately the same probabilities of default. By stressing the latent systemic variable $Z \sim N(0,1)$ one may obtain quantile estimates of the future possible default rate, which is the goal of the regulatory capital calculations.

On the other hand, if we are given a realized default rate p on a specific homogenous portfolio with the known parameters PD and ρ, then the latent (not observable) systemic factor z can be derived from (2) as

$$z = \frac{\Phi^{-1}(PD) - \sqrt{1-\rho}\Phi^{-1}(p)}{\sqrt{\rho}}. \tag{3}$$

If we observe a time series of historical default rates p_t, for $t = 1, \dots, T$, then we can even estimate the parameters PD and ρ if these are unknown. For this purpose, it is useful to formulate the model (2) in the form $p_t = \Phi(\alpha - \beta z_t)$ where $z_t = \left(\alpha - \Phi^{-1}(p_t)\right)/\beta$ is the realized latent systemic factor. Thus, if we assume that the Z-factors are independently drawn from the standard normal distribution, then the unknown parameters can be estimated by maximizing the likelihood function. It can be shown (Yang, 2013) that the maximum likelihood estimates can be simply obtained from the mean (α) and standard deviation ($-\beta$) of the transformed series $\Phi^{-1}(p_t), t = 1, \dots, T$. The correlation and long-term probability of default can be then calculated as follows:

$$\rho = \frac{1}{1+\beta^2}, PD = \Phi\left(\alpha\sqrt{\rho}\right). \tag{4}$$

It is obvious that the assumption of the Gaussian independent realizations of z_t is questionable. Regarding independence, it is certainly important to observe the default rates over non-overlapping time intervals. However, even in this case the series of systematic factors might be autocorrelated and a more general specification should be considered (see e.g. Witzany, 2011).

Regarding the probability distribution of the systematic factor and the idiosyncratic factors, one may generalize the model stipulating that $Z \sim F_1$, $\xi_i \sim F_2$, and $Y_i \sim F_3$, where F_3 the mix of the independent distributions F_1, F_2 given by the parameter ρ in (1). The formula (2) then takes the following form:

$$PD_i(Z) = F_2\left(\frac{F_3^{-1}(PD_i) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right). \tag{5}$$

Given a time series of historical default rates as above, the parameters in the model $p_t = F_2(\alpha - \beta z_t)$ can be again obtained by MLE from the transformed latent factors

$$z_t = \left(\alpha - {F_2}^{-1}(p_t)\right)/\beta$$

and the assumption that z_t are (independently) drawn from F_1. Specifically, the likelihood of the single default rate p_t observation is

$$L(p_t) = \frac{f_2(z_t)}{\beta f_2\left({F_2}^{-1}(p_t)\right)},$$

And so the total log-likelihood is

$$LL(\alpha,\beta) = \sum_t \ln f_2(z_t) - \ln f_2\left(F_2{}^{-1}(p_t)\right) - \ln\beta\,. \tag{6}$$

The correlation parameter ρ that is calculated as in (4) then determines the mixed distribution F_3 which can be used to get the unconditional probability of default $PD = F_3\left(\alpha\sqrt{\rho}\right)$. Witzany (2013) proposes to use the logistic distribution due to the fact that the Logit model (logistic regression) has become an industry standard for credit risk modelling. The empirical study shows that the impact of the logistic distributional assumptions compared to the Gaussian model is dramatic in case of unexpected default rate estimation on high probability levels (such as 99.9% used by the Basel regulation). Another candidate distribution is the Student t-distribution proposed for example in Perederiy (2017).

Next, the single factor model outlined above needs to be further generalized since the goal is to stress rating migration matrices conditionally on economic scenarios as explained in the introduction. Let us assume that there are K rating grades $u = 1, \dots, K$ where K is the absorbing state of default. Our goal is to estimate the probabilities of future rating transitions

$$PD_{uv}(i,t) = \Pr[\mathrm{rat}(i,t) = v|\mathrm{rat}(i,t-1) = u] \tag{7}$$

where $\mathrm{rat}(i,t) = u$ is the rating grade assigned to exposure i at time t. If the transition probabilities are modelled on a homogenous portfolio, then the index i can be dropped. Let us assume that there are historical default and rating transition observations for periods $t = 1, \dots, T$ and our goal is to estimate the forward-looking transition probabilities $PD_{uv}(t|\mathbf{X}_t)$ conditional on a scenario given by a vectors series of macroeconomic variables $\mathbf{X}_t$ for $t = T+1, \dots, T+M$ (where M is the maximal maturity for which the transition probabilities need to be modelled).

We will firstly describe a relatively straightforward generalization of the single-factor model that is often used in practice in the banking sector for the purpose of IFRS 9 or ICAAP ST modelling (Petrov and Rubtsov, 2016), and then consider its refinements and alternative approaches. The key assumption of the generalized single-factor model is that not only the event of default but also the transitions are driven by the factors (1), i.e. dropping the time parameter t the unconditional probability of transition from u to v is

$$PD_{uv} = \Pr\left[b_{u,v+1} < Y_i \le b_{u,v}\right] \tag{8}$$

where $b_{u,v}, v = 1, \dots, K$ are decreasing transition thresholds and $b_{u,K+1} = -\infty$. It is useful to introduce the cumulative transition probabilities in order to express the thresholds from the probabilities:

$$\begin{aligned} PD_{uv+} &= \Pr[\mathrm{rat}(i,t) \ge v|\mathrm{rat}(i,t-1) = u] = \\ &= \Pr\left[Y_i \le b_{u,v}\right] = PD_{uv} + PD_{u,v+1} + \dots + PD_{uK}. \end{aligned} \tag{9}$$

Then $b_{u,v} = \Phi^{-1}\left(PD_{uv+}\right)$ assuming that the distribution of Y_i is standard normal. Therefore, we can express the conditional transition probabilities similarly to (2):

$$PD_{uv+}(Z) = \Phi\left(\frac{\Phi^{-1}(PD_{uv+}) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right). \tag{10}$$

Given the conditional cumulative transition probabilities we can easily obtain the standard transition probabilities, $PD_{uv}(Z) = PD_{uv+}(Z) - PD_{u(v+1)+}(Z)$.

Hence, a relatively simple standard approach can be described as follows:

1. Given a time series of historical default rates $p_t, t = 1, \dots, T$ on a pool of exposures estimate the correlation ρ and calculate the latent historical Z-factors $z_t, t = 1, \dots, T$ based on (1).
2. Use the historical rating transitions to estimate a through-the-cycle (TTC) transition matrix $\langle PD_{uv}^{TTC} \rangle_{u,v=1}^{K}$.
3. Select a vector of appropriate lagged or contemporaneous predictors $\mathbf{X}_t, t = 1, \dots, T$ and build a model macroeconomic model that links z_t and the predictors. The model can be just a regression, VAR, or VEC model, etc.
4. For the desired stress-testing or IFRS9 scenarios estimate the forward looking values of the economic indicators $\mathbf{X}_t^{scen}$ and the implied factors z_t^{scen} (where $t = T + 1, \dots, T + M$) using the model from the previous step. The scenarios definition and macroeconomic predictions can be obtained from an external source (e.g. central bank predictions) or by developing an internal model that is not in the scope of this paper.
5. Adjust the TTC transition matrix is to $\langle PD_{uv}^{scen}(t) \rangle_{u,v=1}^{K}$ conditional on z_t^{scen} for $t = T + 1, \dots, T + M$ using (10). The transition matrices can be then multiplied through to get migration probabilities over longer periods to estimate overall expected credit losses conditional upon the scenario.

This relatively straightforward and elegant approach has a few weaknesses we want to point out and investigate. Firstly, the single-factor default model with the correlation parameter ρ assumes that the portfolio is homogenous in terms of credit risk. However, this is not the case for a portfolio where our aim is to model rating transition probabilities (for example a mortgage portfolio, consumer loans, corporate loans, etc.), and so by definition there must be exposures with various rating. It means that the estimation of one correlation fitting all initial ratings in Step 1 violates the theoretical credit risk homogeneity assumption. A possible remedy is to estimate correlations ρ_u based on time series of rates of defaults conditional on the initial rating u or possibly using all the observations of transitions from u to another rating v.

Secondly, as discussed above, the Gaussian distribution assumption has many technical advantages but does not correspond to the market practice and wide empirical evidence that leads to preference of the logistic distribution. Our proposal is to use the model (5) with the logistic regression not only for stressing of the TTC transition matrix but also for the estimation of the correlation parameter(s), i.e. consistently in the modelling steps 1-5.

Our goal is to compare empirically the results in terms of stressed life-time probabilities of default estimated using the following four alternative models:

1. Gaussian model with a flat correlation (independent on the initial rating),
2. Gaussian model with rating dependent correlations,
3. Logistic model with a flat correlation,
4. Logistic model with rating dependent correlations.

3 Empirical Results

We are going to use the aggregate data from the Moody's Annual Default Study (Corporate Default and Recovery Rates, 1920-2011) as well as the Moody's Default Risk Service (DRS) database containing data for over 11 000 corporate and sovereign entities and spanning from 1970 until 2011. The aggregate data with annual default rates were sufficient to estimate the models with a flat correlation coefficient. The issuer level data were necessary to obtain annual transition rates and estimate rating dependent correlations. The development of the Investment and Speculative Corporate default rates in the period 1920-2011 is shown in Figure 1.

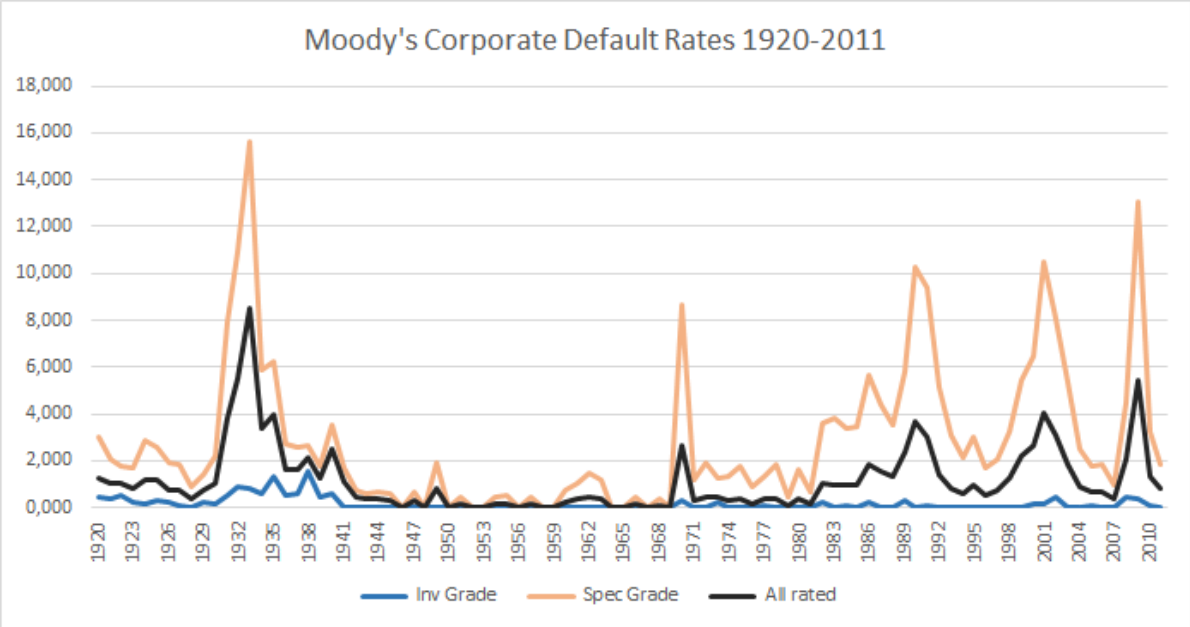


Figure 1. Annual Credit Loss Rates by Letter Rating, 1982-2011 (Source: Moody's Annual Default Study)

Due to low numbers of observed defaults in the period 1942-1981 the default correlations were estimated based on the data from years 1982-2011. The following table summarizes the estimated correlations by the Probit and Logit models and for different rating grade pools using data and the formulas (4) and (6):

Rho	Aa	A	Baa	Ba	B	Caa-C	Inv-Grade	Spec-Grade	All Rated
Probit	6.6%	9.5%	18.5%	21.3%	22.2%	30.7%	11.3%	9.4%	7.969%
Logit	5.9%	36.4%	50.5%	44.4%	36.4%	33.0%	18.6%	17.2%	18.6%

Table 1. Probit and Logit correlations (Rho) depending on the rating grade pools

The correlation was not estimated for the Aaa rating class since the time series contains only zero default rates. Table 1 and Figure 2 below shows that the estimated correlation coefficient significantly depends not only on the model but also on the rating pool on which it is estimated.

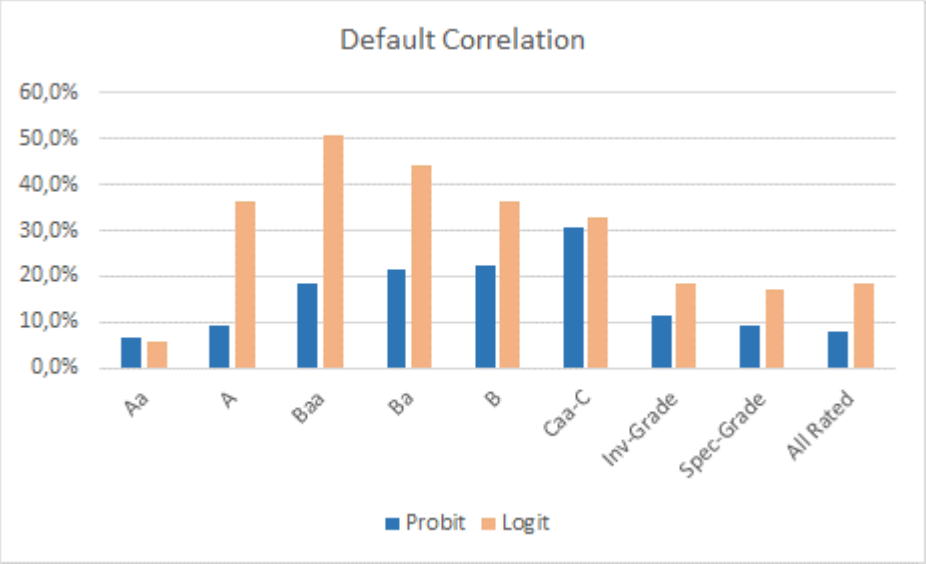


Figure 2. Probit and Logit correlations depending on the rating grade pools

It is not surprising that the "asset correlation" parameter depends on the distribution chosen. However, dependence of the estimate on the rating pool is disturbing since it contradicts the generally accepted model assumptions (with a flat correlation). It seems that the correlation goes up with worse rating for the Probit model and peeks around Baa for the Logit model. On the other hand, the differences between the correlation estimated on the investment and speculative grade pools are not large.

The model assumes that the observed default rates approximate the probabilities of default (conditional expected default rates) which is true in case of an asymptotic portfolio. In case of the Moody's dataset the average observed annual number of defaults in the period 1982-2011 is around 60 and the number of corporate issuers is around 6 000. Therefore, it is not surprising that the default rate series for individual rating grades may contain substantially more noise than for all ratings, investment grade, or speculative grade pools.

To test the phenomenon, we have also combined the time series given by several ratings, e.g. Baa and Ba, or Baa, Ba, B, and estimated the correlation based on the total log-likelihood. The estimated correlation was on the level close to the average of the individual estimates, e.g. 41% for Ba and Baa for the Logit model (the plain average of the two estimates is 43.5%). In fact, it is obvious that the noise will not be diversified away if the likelihood is based on a total of individual ratings default rates, or even based on transitions between individual ratings.

Given the estimated correlations we are going to stress the TTC transition rating matrix based on the Moody's rating transitions observed in 1920-2011 and adjusted for withdrawn ratings (WR) proportionately.

From/To:	Aaa	Aa	A	Baa	Ba	B	Caa	Ca_C	Default
Aaa	90,397%	8,532%	0,868%	0,167%	0,033%	0,001%	0,001%	0,000%	0,000%
Aa	1,265%	89,829%	7,808%	0,791%	0,182%	0,041%	0,006%	0,005%	0,072%
A	0,082%	3,056%	89,932%	5,915%	0,744%	0,128%	0,030%	0,009%	0,102%
Baa	0,044%	0,305%	4,748%	88,255%	5,342%	0,846%	0,141%	0,016%	0,303%
Ba	0,008%	0,093%	0,527%	6,684%	82,843%	7,681%	0,657%	0,076%	1,433%
B	0,007%	0,054%	0,170%	0,660%	6,558%	81,429%	6,337%	0,599%	4,187%
Caa	0,000%	0,023%	0,031%	0,201%	0,871%	9,529%	71,462%	4,291%	13,592%
Ca-C	0,000%	0,029%	0,126%	0,068%	0,577%	3,678%	9,212%	58,606%	27,704%
Default	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	100,000%

Table 2. TTC rating 1Y transition matrix (Source: Moody's Annual Default Study)

Based on a given Z factor representing the stress level and given the correlation parameter ρ we stress the matrix according to (10) with the Gaussian or other distributional assumptions. For example, for $Z = \Phi^{-1}(0.01)$ corresponding to a stress scenario that can take place with 1% probability and the correlation $\rho = 0.08$, we get the (Probit model) stressed 1Y matrix shown in Table 3. The stressed one-year PDs can be seen the last column, for example the rating A stressed PD is 0.578% that is more than five times greater compared to the TTC default rate 0.102%.

From/To:	Aaa	Aa	A	Baa	Ba	B	Caa	Ca_C	Default
Aaa	74,985%	20,673%	3,294%	0,820%	0,205%	0,008%	0,008%	0,000%	0,009%
Aa	0,127%	76,231%	19,210%	2,935%	0,816%	0,201%	0,033%	0,028%	0,419%
A	0,004%	0,428%	80,027%	15,395%	2,808%	0,575%	0,143%	0,042%	0,578%
Baa	0,002%	0,022%	0,817%	80,233%	13,808%	3,003%	0,574%	0,069%	1,474%
Ba	0,000%	0,004%	0,046%	1,337%	73,128%	17,717%	1,997%	0,241%	5,530%
B	0,000%	0,002%	0,011%	0,066%	1,344%	70,683%	13,230%	1,467%	13,196%
Caa	0,000%	0,001%	0,001%	0,013%	0,093%	2,253%	58,394%	6,954%	32,290%
Ca-C	0,000%	0,001%	0,007%	0,005%	0,056%	0,633%	2,683%	43,858%	52,757%
Default	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	100,000%

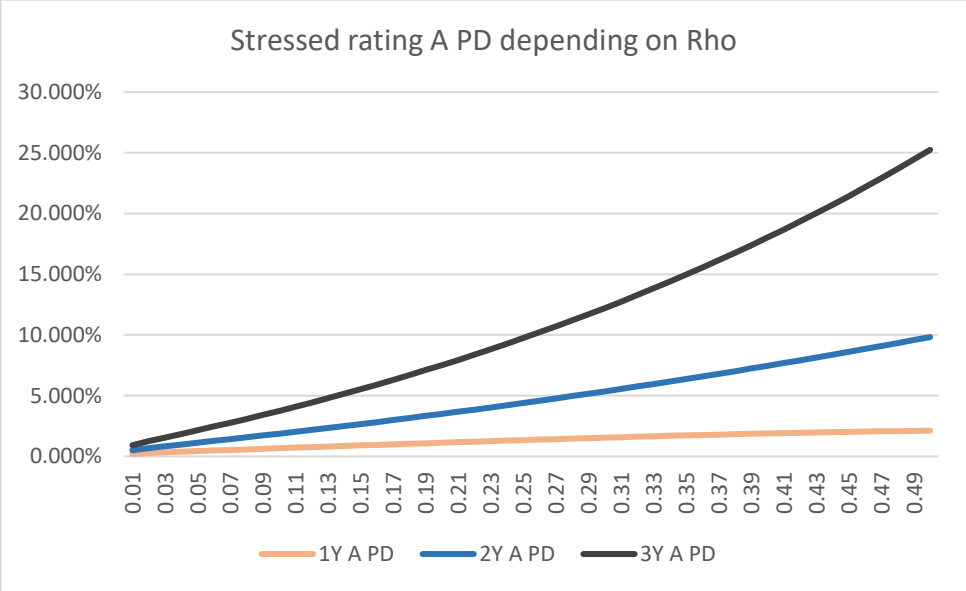
Table 3. Stressed 1Y transition matrix

Generally, we can assume a different level of stress given by Z for the second year, third year, and so on, and obtain the multi-year transition matrices by multiplication. Assuming, for simplicity, the same level of stress for the second and third year given by the factor $Z = \Phi^{-1}(0.01)$ and the same correlation factor $\rho = 0.08$ we get the stressed 3Y correlation matrix shown in Table 4 as the third power of the 1Y matrix (Table 3).

From/To:	Aaa	Aa	A	Baa	Ba	B	Caa	Ca_C	Default
Aaa	42,222%	35,510%	15,150%	4,703%	1,452%	0,406%	0,091%	0,024%	0,442%
Aa	0,220%	44,553%	35,298%	12,431%	3,957%	1,357%	0,312%	0,080%	1,792%
A	0,008%	0,796%	51,753%	29,833%	9,957%	3,558%	0,853%	0,154%	3,087%
Baa	0,003%	0,051%	1,602%	52,393%	24,597%	10,693%	2,587%	0,390%	7,684%
Ba	0,000%	0,010%	0,113%	2,412%	40,055%	27,801%	7,399%	1,152%	21,058%
B	0,000%	0,004%	0,024%	0,163%	2,144%	36,454%	16,803%	3,080%	41,329%
Caa	0,000%	0,001%	0,004%	0,027%	0,196%	2,952%	20,783%	5,582%	70,454%
Ca-C	0,000%	0,001%	0,009%	0,011%	0,083%	0,763%	2,280%	8,730%	88,122%
Default	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	100,000%

Table 4. Stressed 3Y transition matrix

In this way we can easily analyze dependence of the stressed PD on the correlation parameter and the initial rating grade. Figure 3 shows a significant dependence of the 3Y PDs on the correlation parameter. For example, if the portfolio was homogenous rated A with 3Y maturity, the ICAAP stress test capital requirement would double if the default correlation parameter was 16% instead of 8%.



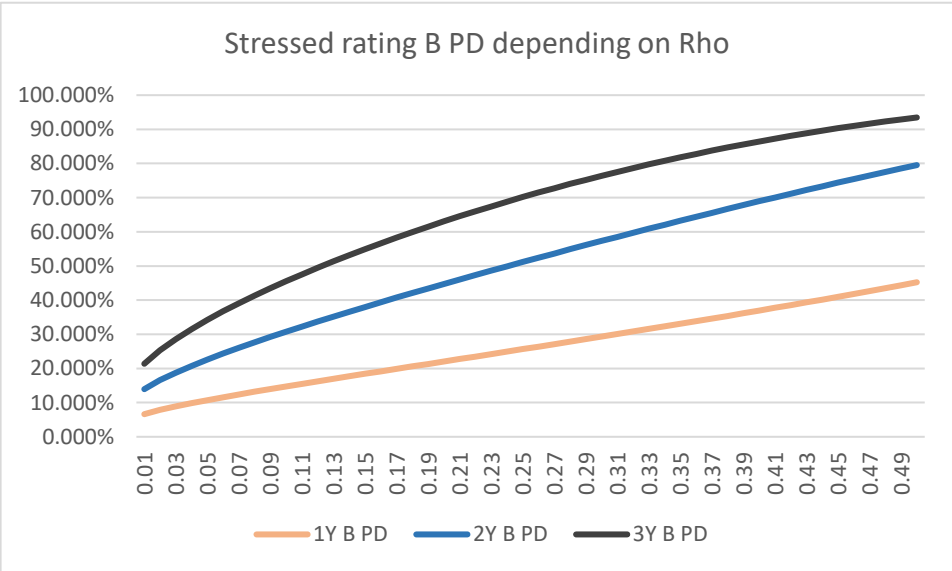


Figure 3. Rating A and B stressed PDs depending on the correlation and maturity

The sensitivity of stressed PDs on the correlation parameter and variability of the estimated correlation depending on the method chosen shown in Table 1 creates a significant model risk. Figure 4 shows the difference between the flat correlation stressed 3Y PDs estimated for all ratings (approximately 8%) and the correlations depending on the rating grades given in Table 1 for the Probit model. For example, if the portfolio was homogenous Baa, the stressed 3Y PD would be 124% higher if we used the rating dependent correlation estimate (21.3%) compared to the flat correlation!

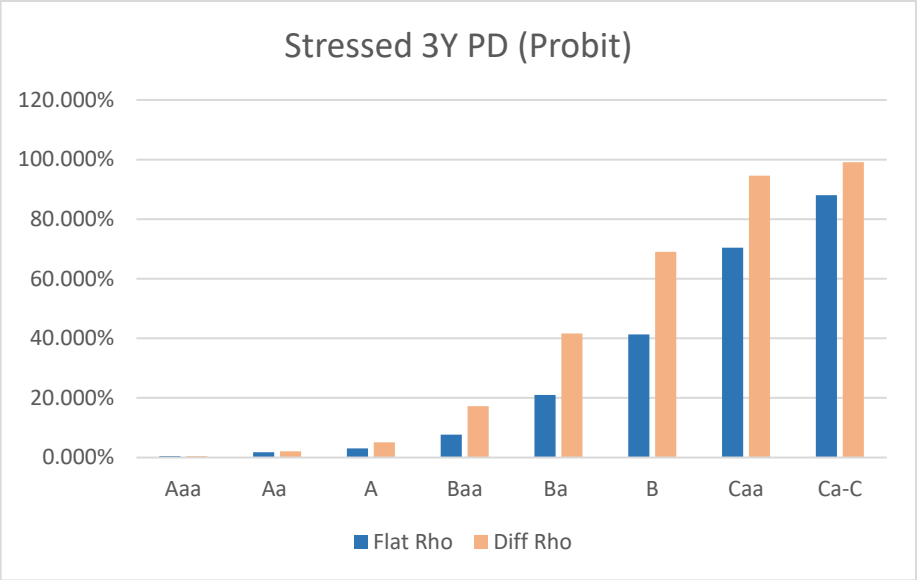


Figure 4. Probit model stressed 3Y PDs calculated using the estimated flat versus rating dependent correlation

The differences are even more dramatic for the Logit model (Figure 5) where, for example, the Baa rating stressed PD based on the rating dependent correlation parameter (50.5%) is more than 230% higher relative to the stressed PD based on the flat correlation (18.6%).

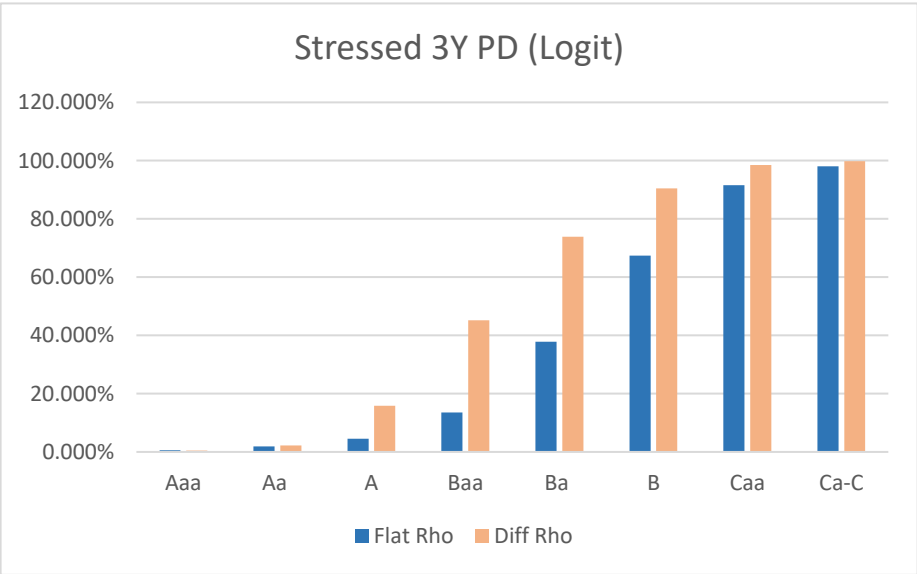


Figure 5. Logit model stressed 3Y PDs calculated using the estimated flat versus rating dependent correlation

Figure 6 finally shows that there is also a significant difference between Probit and Logit estimations even if we use rating dependent PDs. In case of Baa, the Logit model stressed PD is 163% higher than the Probit model based stressed PD.

Altogether, if we compare the current practice, i.e. stressing of PDs based on the Probit model with a flat correlation, and the more realistic Logit model with rating dependent correlations, then in case of Baa 3Y the estimated stressed PD and the corresponding stress-test capital would be, according to our empirical results, almost six-times higher!!!

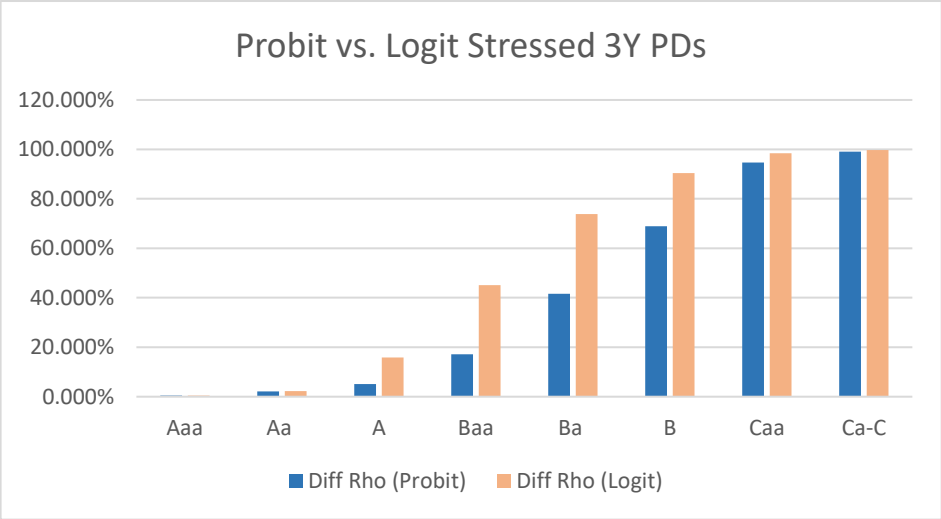


Figure 6. Probit versus Logit stressed PDs (with rating dependent correlations)

The IFRS 9 provisioning standard requires banks to identify expected forward looking baseline macroeconomic scenarios that need to be translated into series of Z-factor values in the context of this class of models. In addition, banks need to identify at least two additional scenarios – an adverse and an optimistic scenario with appropriate probability weights. The final IFRS 9 expected credit loss (ECL) is calculated as the probability weighted average of ECLs conditional on the alternative scenarios. The different level as well as the different convexity of the curves in Figure 7 then apparently cause significant differences between the estimation based on the flat and rating dependent correlations.

For example, if the scenarios were given by $Z_{baseline} = -1$, $Z_{adverse} = -2.15$, $Z_{optimistic} = 0.15$ with probability weights 50%, 25%, 25% (here we assume that the adverse/optimistic scenarios Z-factors differ from the baseline Z-factor by 1.15 corresponding to 12.5% probability level), then the IFRS 9 conditional 3Y PD based on the flat (Probit model) correlations is 3.03% while the rating dependent conditional PD is 4.76%. Therefore, the simplifying flat PD lifetime ECL would underestimate the more precise provisioning calculation by more than 35%!

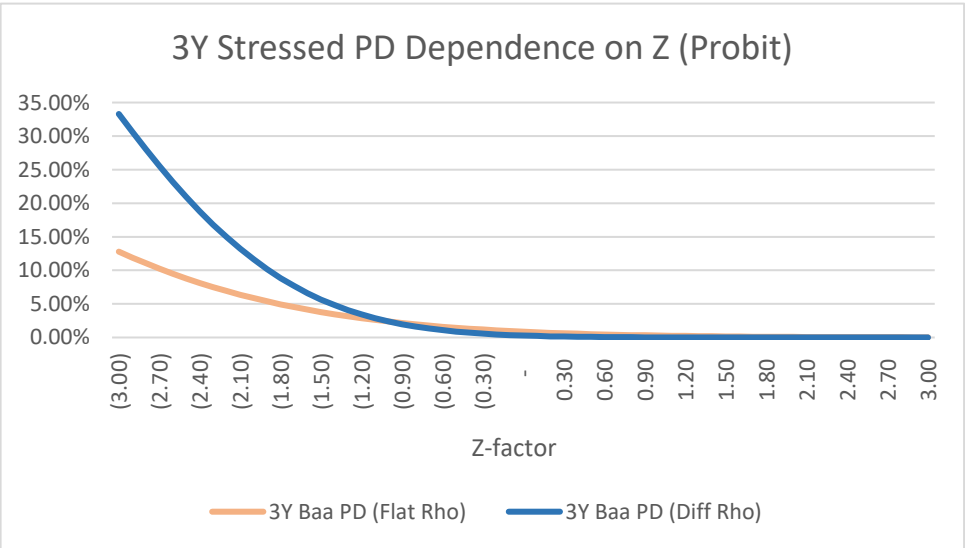


Figure 7. Dependence of the 3Y stressed Baa rating grade PD on the Z-factor (Probit model)

Finally, let us compare the IFRS 9 ECL calculations for the Probit and Logit models with rating dependent PDs. Figure 8 indicates that in the case of Logit model there might be an even more significant difference between the IFRS 9 provisions based on the flat correlation and rating dependent correlation estimates.

To compare the ECL calculated in the context of the Logit model with the example above we need to use Q-Q transformed Z values, $\tilde{Z}_{baseline} = -1.67$, $\tilde{Z}_{adverse} = -4.13$, $\tilde{Z}_{optimistic} = 0.24$. Then IFRS 9 conditional 3Y PD based on the flat correlations and the Logit model is 3.62% while the rating dependent conditional PD is 8.31%. Therefore, the simplifying flat PD lifetime ECL would underestimate the more precise calculation by more than 56%!!! In addition, if we compare the standard approach to IFRS 9 modeling, i.e. the Probit model with flat correlations (PD = 3.03%) and the Logit model with rating dependent correlations (PD = 8.31%) then the error more than 63% relative to the more precise estimation.

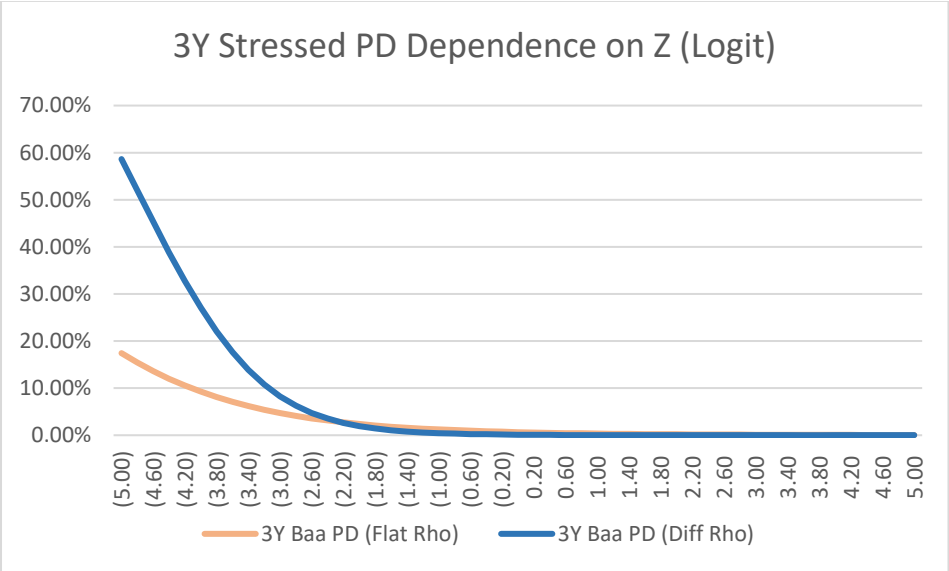


Figure 8. Dependence of the 3Y stressed PD on the Z-factor (Logit model)

4 Conclusion

The main contribution of this paper is a proposed general methodology allowing to stress rating transition matrices with different distributional assumptions, and an empirical study comparing alternative modelling approaches. We are not aware of any such empirical study in the literature. The importance of the matrix stress testing methodology is given by its applications to the ICAAP and IFRS 9 calculations.

The key parameter that can be called "the default or asset correlation" depends on the distributional assumption and may be, in practice, estimated in different ways. Our empirical study demonstrates that the stressed PD estimates and the related ICAAP/IFRS 9 calculations significantly depend on the correlation parameter. In addition, we show that there are large differences between the flat correlation parameter estimated on a pool including all rating grades and the rating depend correlations estimated on individual rating grade pools. While the former approach is quite common in practice, when product pool dependent correlations are

estimated, we argue that the latter approach with rating dependent correlations is more consistent since the theoretical model assumes a PD homogenous portfolio. The dependence of the estimated correlations on the initial rating turns out to be strong both in the context of the Probit and Logit models. There is also an additional significant dependence on the distributional assumptions.

The correlations depending on the rating classes could be compared to the phenomenon of the correlation smile known in the context of credit derivatives valuation. In this analogy, a possible goal of the further research might be identification of a model and distributional assumptions that better fit the empirical data and removes the correlation smile, i.e. allows to stress test consistently the probabilities of default with a single correlation parameter. The empirical study has shown that the correlation smile issue is not unfortunately resolved applying the Logistic distribution, even though it appears to be more appropriate than the Gaussian distribution for credit risk modelling. Therefore, other distributional and model assumptions, including copula dependence models, need to be investigated.

Regarding the impacts to ICAAP and IFRS 9 calculations, it turns out that the standard rating transition matrix stress testing approach (Probit model with a flat correlation) might underestimate the PDs and the ICAAP ST in a more precise approach (Logit model with rating depend correlations) many times. The same effect applies to IFRS 9 scenario conditional ECL calculation where the standard approach might underestimate a more precise calculation more than two times. Therefore, we have demonstrated that there is a significant model risk in both ICAAP and IFRS 9 calculations based on the concept of rating transition matrices that are stressed conditional on various macroeconomic scenarios. Our recommendation for banks and regulators, based on the empirical study, is to use the more consistent and conservative rating dependent approach with heavy-tail distributional assumptions in order to estimate default correlations for the purpose of ICAAP and/or IFRS 9 calculations.

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