Profitability of Trading in the Direction of Asset Price Jumps – Analysis of Multiple Assets and Frequencies

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Abstract

The profitability of a trading system based on the momentum-like effects of price jumps was tested on the time series of 7 assets (EUR/USD, GBP/USD, USD/CHF and USD/JPY exchange rates and Light Crude Oil, E-Mini S&P 500 and VIX Futures), in each case for 7 different frequencies (ranging from 1-Minute to 1-Day), over a period of more than 20 years (for all assets except for the VIX) ending in the second half of 2015. The proposed trading system entered long and short trades in the direction of price jumps, for the closing price of the period in which the jump occurred. The position was held for a fixed number of periods that was optimized on the in-sample period. Jumps were identified with the non-parametric *L*-Estimator whose inputs (period used for local volatility calculation and confidence level used for jump detection) were also optimized on the in-sample period. The proposed system achieved promising results for the 4 currency markets, especially at the 15-minute and 30-minute frequencies at which 3 out of the 4 tested currencies turned profitable (with highest profits achieved by USD/CHF, followed by EUR/USD and GBP/USD), with the profits totalling up to 30-50% p.a. in the case of a high-leverage scenario, or 15-25% in the case of a low-leverage scenario. Additionally, the 5-minute frequency turned profitable for USD/CHF and the 4-hour frequency for GBP/USD, while the 1-minute frequency was unprofitable in all cases due to the commissions and the 1-day frequency contained too few jumps to make any conclusions. As for the futures markets, the system achieved profits only on the Light Crude Oil market, on the frequencies of 1-hour, 4-hour and 1-day, with the profits totalling up to 20% p.a. in the case of high leverage or 10% p.a. in the case of low leverage. For USD/JPY, E-Mini S&P 500 Futures and VIX Futures the system achieved mostly a loss. We attribute this (in the latter two cases) to the effect of a rising market risk premium in the case of negative jumps, going against the jump-momentum effect used by the system.

AMS/JEL classification: C14, C58, G11, G14, G17

Keywords: Asset price jumps, *L*-Estimator, High-frequency trading, Momentum trading, Investment strategy

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Introduction

It is an established fact in financial econometrics that the behaviour of financial time series contains two sources of variability, a continuous, stochastic volatility component, and a discontinuous, jump component. Jumps, representing large discontinuous movements of asset prices, significantly increase the tails of the short-horizon asset return distribution, with significant implications for tasks such as option pricing, VaR estimation, market making or quantitative trading.

Numerous studies have analysed the jump dynamics, identifying effects such as absolute-value size dependency (Fičura, Witzany 2016), jump clustering and contagion effects caused by the jump self-exciting and cross-exciting effects (Ait-Sahalia et al. 2014 or Fulop, Li and Yu 2014), positive impact of jumps on the future asset price volatility (Corsi, Pirino, Reno 2010), etc.

With the increased availability and quality of high-frequency financial data and with the development of efficient non-parametric jump estimators based on the asymptotic theory of power variations, studies have emerged, finding that jumps may even have an impact on the direction of future asset price returns, when studied at high frequencies, carrying a potentially profitable trading signal. Among recent studies analysing the after-jump behaviour of asset prices is the study of Behfar (2016) finding evidence for long-memory behaviour of the S&P 500 Index after the price jumps, or the study of Novotny, Petrov and Urga (2015), analysing the profitability of after-jump trading on the 5-minute frequency of the foreign exchange rate time series, finding that for Euro, Yen and Rand, it is possible to achieve profit with this strategy, even in the presence of a bid-ask spread.

As the jumps often coincide with macroeconomic news announcements, it is possible to link the momentum-like after-jump effect to the delayed reaction of asset prices to macroeconomic news announcements. The evidence on whether the prices react to macroeconomic news announcements efficiently or if their reactions are delayed (or if the possibly overshoot), is rather mixed, with studies of Brazys and Martens (2014) and Brazys, Duyvesteyn and Matens (2015) indicating a possibly delayed reaction for the bond markets to macroeconomic news announcements, while the study of Andersen et. al (2006) found no evidence of a statistically significant reaction to macroeconomic news beyond the first 5 minutes after the news announcements, in the bond, stock and the foreign exchange markets.

As the possible existence of a predictable after-jump behaviour violates the efficient market hypothesis, while at the same time being potentially very interesting for investors and speculators, we perform a detailed analysis of this phenomenon for multiple assets on multiple frequencies, in order to either confirm or reject its existence and to further elaborate on the potential profitability of the trading signals that it offers.

In the performed analysis, we study the after-jump behaviour on 7 different markets (EUR/USD, GBP/USD, USD/CHF and USD/JPY exchange rates, and the S&P500, VIX and Light Crude Oil futures prices) in a time-period of up to two decades, depending on the data availability for the given asset. The jumps in the time series are identified with a non-parametric *L*-Estimator proposed by Lee and Mykland (2008), which identifies price jumps at the exact time when they occur and it is thus particularly well suited for trading purposes. The analysis is performed for each of the analysed assets on 7 different frequencies (ranging from 1 minute to 1 day), in order to analyse at what frequency do the markets start to get efficient (i.e. the jumps stop to have predictive power with regards to the future returns at that frequency).

An after-jump trading system is proposed and its profitability is evaluated, in a methodologically robust way that avoids the risks of the forward-looking bias and selection bias. The whole data

sample is divided into an in-sample and out-sample period (first vs. second half of the available data), with the in-sample period being used to fine-tune the parameters of the proposed trading algorithm, such as the period used for the local volatility calculation in the *L*-Estimator, the confidence level used for the jump identification, and the horizon during which the positions are held. The out-sample period (covering the most recent half of the data for each of the assets) is then used to evaluate the performance of the fine-tuned models with regards to their profitability and risk, in the presence of transaction costs.

The rest of the study proceeds as follows. In the first section we explain the notion of realized bipower variation (used for local volatility estimation) and the *L*-Estimator used for non-parametric and model-free identification of jumps in the financial time series. In the second section, the dataset is presented and statistics about the identified jumps in the time series are discussed. In the third section, the after-jump trading strategy is proposed and its results are evaluated. Finally, in the last section, the results of the study are summarized and conclusions about the profitability of the proposed systems are made.

Non-parametric jump identification

As the jumps in financial time series are inherently unobservable (i.e. we do not know If a given large return was a jump or if it was caused by the continuous volatility component), it is necessary to estimate them. This was traditionally done with the Bayesian estimation methods (for their comparison with an non-parametric approach see Ficura and Witzany 2016). With the increased availability of high-frequency financial data, however, a new class of jump estimators was proposed, utilizing high-frequency returns and the asymptotic theory of power variations. These methods exhibit high accuracy in simulation studies, as well as in empirical studies, and they also have the benefit of being model-free (i.e. the jump estimators are valid for a wide variety of possible price processes, so we do not have to assume some specific process for them model to give accurate results).

In our study the jumps in the analysed time series are identified with the *L*-Estimator proposed by Lee and Mykland (2008). The estimator uses returns normalized by the local volatility (estimated with the realized bipower variation) to identify if the given return was caused by a jump or not. Specifically, the method compares the size of the local volatility normalized returns with a given quantile of the distribution of the expected maximum of these normalized returns in a time series of the given length, with the distribution derived under the assumption of no jumps in the time series. If the absolute size of the return is greater than the given quantile, it is identified as a jump. Important benefits of the *L*-Estimator compared to some other well-known non-parametric estimators such as the Z-Estimator, proposed by Barndorff-Nielsen and Shephard (2004), is, that it can be applied to any frequency (although the high-frequency data lead naturally to higher accuracy jump estimates) and that it identifies the jumps at the exact time when they occur (instead of just determining if a jump occurred during a given time-period as the Z-Estimator does).

In order to explain the logic of the *L*-Estimator and of the bi-power variation used to compute the local volatility, we first define a general stochastic-volatility jump-diffusion process governing the evolution of the logarithmic returns of the price of a given asset:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + j(t)dq(t) \qquad (1)$$

Where $p(t)$ is the logarithm of the asset price, $\mu(t)$ is the instantaneous drift rate, $\sigma(t)$ is the instantaneous volatility, $W(t)$ is a Wiener process, $j(t)$ is a process determining the jump sizes and $q(t)$ is a counting process determining the jump occurrences.

It is worth noting that in line with the model-free nature of the employed approach, the process in equation (1) represents just a general specification of a jump-diffusion stochastic process of the price evolution, with the sub-processes governing $\mu(t)$, $\sigma(t)$, $j(t)$ and $q(t)$, being free to attain a wide variety of different forms.

The total variability of the process governing the asset price evolution over the period between $t-k$ and t can be expressed with its *quadratic variation* as follows:

$$QC(t-k,t) = \int_{t-k}^{t} \sigma^2(s)ds + \sum_{t-k\leq s<t} \kappa^2(s) \qquad \text{Eq. 2}$$

Where $QC(t-k,t)$ denotes the quadratic variation for the period between $t-k$ and t and $\kappa(s) = j(t)I[q(t)=1]$ with $I(.)$ denoting an indicator function. The first term on the right side of the equation, $\int_{t-k}^{t} \sigma^2(s)ds$, represents the continuous component of the asset price variability, and is called *integrated variance*, while the second term, $\sum_{t-k\leq s<t} \kappa^2(s)$, represents the aggregated impacted of jumps on the price variability over the given period of time, and is called *jump variance*. It is thus possible to rewrite the equation as follows:

$$QC(t-k,t) = IV(t-k,t) + JV(t-k,t) \qquad \text{Eq. 3}$$

Where $IV(t-k,t)$ is the integrated variance and $JV(t-k,t)$ is the jump variance.

As already mentioned, the *L*-Estimator estimates jumps by comparing the size of the asset returns with the local volatility corresponding to the given return. In order to define the local volatility (as a measure of the continuous price variability) for each time-point in the time series, the authors use integrated variance over the last k periods, which can be consistently estimated, even in the presence of jumps, by the realized bi-power variation defined as follows:

$$\sigma_{BV}^2(i) = BV(i-K,i-1) = \frac{1}{K-2}\sum_{j=i-K+2}^{i-1}|r(j)||r(j-1)| \qquad \text{Eq. 4}$$

Where $\sigma_{BV}^2(i)$ is the local variance estimate for day i, $r(j)$ is a logarithmic return at period j, defined as $r(j) = p(j) - p(j-1)$ with $p(j)$ being the logarithm of the price at day j, and K is the period (i.e. number of days) used for the local volatility estimation. $BV(i-K,i-1)$ is then the realized bipower variation over the period between $i-K$ and $i-1$, which converges, with increasing frequency of the returns used for its calculation, to the integrated variance $IV(i-K,i-1)$.

The *L*-Estimator can then be defined as follows:

$$L(i) = \frac{r(i)}{\sigma_{BV}(i)} \qquad \text{Eq. 5}$$

Where $L(i)$ is the *L*-Statistics, $r(i)$ is the return in the given time-period and $\sigma_{BV}(i)$ is the local volatility (i.e. the square root of the local variance $\sigma_{BV}^2(i)$).

The method of jump identification based on the *L*-Estimator uses the known distribution of the appropriately normalized maximum value of the *L*-Estimator in a time series A_n of a length n, with the distribution derived under the assumption of no jumps in the time series. The normalized maximum can be expressed as:

$$\xi = \frac{max_{i \in A_n}|L(i)| - C_n}{S_n} \qquad \text{Eq. 6}$$

Where A_n denotes a set of all periods in the analysed time series, $i \in \{1,2,\dots,n\}$, and C_n and S_n are constants expressed as follows:

$$C_n = \frac{[2\log(n)]^{1/2}}{c} - \frac{\log(\pi)+\log[\log(n)]}{2c[2\log(n)]^{1/2}} \qquad \text{Eq. 7}$$

$$S_n = \frac{1}{c[2\log(n)]^{1/2}} \qquad \text{Eq. 8}$$

And c is a constant equal to $c = \sqrt{2}/\sqrt{\pi} \approx 0.7979$.

To identify jumps in the time series it is necessary to utilize the known distribution of the maximum ξ in a time series of length n that does not contain jumps. The distribution of the maximum ξ is:

$$P(\xi \leq x) = exp(-e^{-x}) \qquad \text{Eq. 9}$$

Jumps can then be identified as the normalized values of $L(i)$, with the normalization performed based on Eq. 6, that are larger than a given quantile of ξ. I.e. larger than some sufficiently high quantile α of the expected maximum value ξ observed in a time series of length n under the assumption of no jumps in the time series.

The jump estimation approach using the *L*-statistics has two meta-parameters, the period *K*, used for the local volatility calculation, and the quantile α, used as a confidence level for the jump detection. Different values of *K* are proposed by Lee and Mykland (2008) for different frequencies of the time series from which the jumps are identified. In our study the values of *K* and α will both be optimized on the in-sample period, in order to find out, what values lead to jumps with greatest predictive power with regards to the future returns. Different values of *K* and α will also be tested in section two, in order to see how the number of identified jumps in the time series depends on the choice of these values.

Dataset for the jump analysis

The dataset used in the jump analysis contains time series (with different frequencies) for 4 foreign exchange rates (EUR/USD, GBP/USD, USD/JPY and USD/CHF) and 3 futures contracts (E-Mini S&P 500 Futures, VIX Futures and Light Crude Oil Futures. The currency exchange rate dataset ranges from 1.11.1999 to 12.6.2015. The futures dataset period differs for each of the underlying assets, for E-Mini 500 ranging from 11.9.1997 to 4.12.2015, for VIX ranging from 26.3.2004 to 2.12.2015 and for Light Crude Oil ranging from 2.1.1987 to 4.12.2015. The provider of the foreign exchange rates dataset is forexhistorydatabase.com, while the provider of the futures dataset is tickmarketdata.com. The analysis is performed on 7 different frequencies (1-minute, 5-minute, 15-minute, 30-minute, 1-hour, 4-hour and 1-day). The purpose of using multiple frequencies is to evaluate whether the markets are inefficient (with regards to the after-jump behaviour) only on the highest frequencies (as other studies have found) or if it possible to observe inefficiency even on the lower frequencies. It will also enable us to determine what frequency is most profitable for trading based on the jump signals, as while the price behaviour on the high-frequencies may be less efficient, the jump identification method will naturally identify more, smaller, jumps on the high frequencies, and it is not clear whether the predictive power of these small jumps is high enough in order to cover the high transaction costs of trading them. Therefore, we expect the highest profits to occur rather at middle frequencies.

The data were cleaned of weekend gaps and roll-over returns (for the futures series) as these may generate false jumps in the time-series corresponding to different issues than the rapid discontinuous price movements that we want to study (and that we believe may carry a profitable trading signal)

Table 1 and Table 2 show the identified numbers of jumps for all of the analysed time series, at different frequencies and with different parameters used for the jump estimation (different numbers of periods *k* for the local volatility calculation and different probabilities of confidence *p* for the jump identification). The same combinations of parameter values will be used in the next section in order to fine-tune the proposed after-jump trading system in the in-sample period, with the fine-tuned systems being subsequently applied to the out-sample period and their results evaluated. Specifically, we work with the values of $k=\{4,8,16,32,64,128,256\}$ and with the values of $p=\{0.9,0.95,0.99,0.999\}$.

We can see from Table 1 and Table 2 the following 3 tendencies. Firstly, the higher the frequency of the returns, the higher is the number of identified jumps This is entirely with accordance with our expectations as on the higher frequencies it is easier ty identify all potentially small discontinuous changes of the price, while at the higher frequencies most of these discontinuous changes will get lost in the overall price variability and become indistinguishable from the continuous volatility component.

Second tendency that can be observed is that with higher confidence levels used for jump identification, lower number of jumps is identified. This is obviously also in accordance with our expectations and it actually must be necessarily true in all of the cases.

The third tendency is that with higher number of periods used for local volatility estimation, less jumps are identified. This is, from our opinion, caused by the multifractal character of the continuous price volatility, causing the volatility to exhibit clustering at all existing frequencies. At the minute frequency there are small clusters, ranging for only several minutes, while at hourly, daily or even weekly frequencies there are similar clusters, but lasting in the horizon of several hours, days or weeks. If the parameter *k* for the local volatility estimation is then set too low, low local volatility is estimated every time, when the volatility cycle at the given frequency reaches a minimum, with the subsequent increase in volatility being miss-classified by the estimator as jump. Due to this cyclical character of the volatility process, we would advise against too low number of periods used for local volatility estimation, as the number of identified jumps may then rise to unrealistically high numbers, as can be seen for the highest frequencies and low values of *k*. Additionally, the intraday seasonality of the volatility leads to the same effect in this regard. During the night, volatility is low, while with the opening of the different world markets (i.e. the beginning of the Asian, European or US trading session) the volatility often spikes, which may lead to an identification of a jump although the return was caused by the continuous volatility component. While these issues may complicate the identification of the real price jumps, they are of no so much concern in our study as we simply set he value of *k* in order to maximize the in-sample profit of the proposed after-jump trading strategy and do not care so much whether some proportion of the jumps is misclassified or not. As long as the "jumps" carry predictive power with regards to the future returns, they are of interest to us.

Table 1 – Number of jumps identified by the L-Estimator with p={0.9,0.95,0.99,0.999} and k={4,8,16}

analyzed series		k=4				k=8				k=16			
currency	**freuency**	90%	95%	99%	99,90%	90%	95%	99%	99,90%	90%	95%	99%	99,90%
EUR/USD	1-Day	262	237	203	165	43	40	28	21	16	13	8	6
	4-Hour	2023	1929	1739	1548	523	481	382	294	260	222	178	124
	1-Hour	7579	7371	6842	6292	2010	1863	1593	1266	1319	1218	1045	769
	30-Minute	14992	14608	14057	13184	3231	3017	2580	2065	1799	1660	1369	1040
	15-Minute	33124	32599	31484	30192	4824	4457	3818	3113	2656	2415	2017	1580
	5-Minute	127345	126774	123634	121385	14089	13586	11660	9762	4530	4132	3487	2732
	1-Minute	981945	975581	971397	965813	135792	132109	120681	117001	24594	23123	19075	15891
GBP/USD	1-Day	256	231	193	164	45	36	26	16	9	6	5	4
	4-Hour	2003	1926	1747	1523	465	415	321	240	241	213	166	119
	1-Hour	6650	6464	5980	5426	1865	1732	1445	1146	1242	1130	908	698
	30-Minute	13520	13122	12478	11677	2870	2661	2211	1713	1645	1514	1244	962
	15-Minute	29175	28639	27421	26056	4234	3890	3243	2580	2236	2038	1688	1311
	5-Minute	112693	112048	108580	106034	12843	12213	10421	8614	4220	3856	3192	2420
	1-Minute	877676	869709	864615	857886	116259	112074	102370	97629	26030	24567	20498	17016
USD/CHF	1-Day	267	245	208	174	52	45	30	23	13	11	10	7
	4-Hour	2055	1979	1800	1573	566	513	423	322	291	259	206	143
	1-Hour	7184	6949	6437	5888	2023	1873	1571	1243	1363	1254	1024	785
	30-Minute	14555	14102	13463	12534	3222	2988	2543	2045	1852	1701	1406	1095
	15-Minute	31312	30777	29577	28201	4699	4379	3715	3008	2558	2350	1916	1484
	5-Minute	118943	118288	115013	112577	14002	13429	11641	9842	4509	4148	3483	2712
	1-Minute	916148	909580	904689	898401	132838	129172	118731	114292	29305	27803	23353	20123
USD/JPY	1-Day	273	262	231	194	54	49	36	18	28	23	20	12
	4-Hour	1790	1722	1578	1394	473	430	347	269	256	221	170	122
	1-Hour	7367	7206	6745	6248	1723	1589	1324	1063	902	826	686	531
	30-Minute	15749	15324	14729	13858	3010	2799	2387	1881	1486	1369	1136	851
	15-Minute	34792	34268	33229	31998	4916	4552	3926	3211	2339	2131	1749	1376
	5-Minute	134606	134088	131078	129074	15923	15436	13539	11657	4635	4311	3620	2799
	1-Minute	995698	989501	985392	980004	156487	153115	141689	138036	32380	31149	25881	22786

Table 2 – Number of jumps identified by the L-Estimator with p={0.9,0.95,0.99,0.999} and k={32,64,128,256}

analyzed series		k=32				k=64				k=128				k=256			
currency	**freuency**	90%	95%	99%	99,90%	90%	95%	99%	99,90%	90%	95%	99%	99,90%	90%	95%	99%	99,90%
	1-Day	7	4	4	3	4	4	3	3	5	4	4	3	5	4	4	4
	4-Hour	151	132	97	65	130	110	79	47	114	100	71	38	108	95	63	39
	1-Hour	712	652	518	379	600	542	432	311	461	419	331	240	451	408	319	220
EUR/USD	30-Minute	1529	1394	1161	885	940	852	691	543	887	813	665	502	781	732	586	435
	15-Minute	2031	1845	1509	1172	1994	1835	1524	1165	1386	1248	1026	814	1357	1239	1014	796
	5-Minute	3548	3280	2724	2150	3496	3199	2677	2119	3598	3316	2761	2164	3503	3223	2725	2164
	1-Minute	11243	10472	8891	7047	9044	8397	7093	5725	9080	8424	7103	5781	9694	8978	7688	6188
	1-Day	5	5	3	3	4	4	3	3	6	5	3	3	8	6	3	2
	4-Hour	133	113	80	45	103	90	62	38	80	72	48	28	81	61	48	24
	1-Hour	522	467	379	254	462	413	309	213	351	311	245	165	325	292	226	151
GBP/USD	30-Minute	1377	1256	1030	759	723	640	504	372	685	618	490	354	573	516	403	298
	15-Minute	1796	1644	1304	1013	1684	1527	1237	937	1020	915	737	550	1022	925	737	563
	5-Minute	3225	2929	2408	1853	3155	2904	2357	1794	3220	2953	2416	1879	2796	2547	2070	1608
	1-Minute	12261	11403	9602	7597	9572	8827	7345	5796	9214	8494	7131	5672	9695	8945	7515	5992
	1-Day	9	6	6	6	9	9	7	6	10	10	8	7	9	9	8	7
	4-Hour	172	150	113	76	142	126	88	58	119	103	78	49	119	97	66	44
	1-Hour	691	619	511	379	593	542	442	316	477	435	352	250	452	416	328	253
USD/CHF	30-Minute	1587	1452	1199	907	975	877	736	555	915	844	688	521	798	723	589	464
	15-Minute	2012	1844	1502	1151	1972	1808	1526	1190	1345	1252	1036	794	1346	1230	1018	798
	5-Minute	3588	3283	2721	2120	3508	3202	2668	2102	3698	3416	2798	2180	3582	3323	2735	2127
	1-Minute	13205	12307	10537	8416	10132	9287	7835	6163	9669	8911	7487	5932	10283	9486	8001	6431
	1-Day	19	14	13	7	22	19	14	9	18	17	10	7	19	15	8	5
	4-Hour	168	150	110	74	129	105	85	63	117	105	76	57	116	102	72	48
	1-Hour	593	543	440	337	482	439	354	270	417	381	299	233	415	372	293	217
USD/JPY	30-Minute	1054	957	792	615	875	799	646	497	792	719	576	452	727	651	525	411
	15-Minute	1677	1551	1283	960	1479	1354	1104	862	1282	1169	961	769	1243	1142	938	748
	5-Minute	3269	3014	2502	1965	3127	2866	2379	1873	2919	2700	2248	1774	2959	2744	2306	1822
	1-Minute	11989	11263	9731	7677	8588	7953	6805	5386	8066	7442	6264	5062	8258	7720	6595	5341

After-Jump trading strategy analysis

In this section the profitability of an after-jump trading system is analysed. The system utilizes the momentum-like effect of jumps, i.e. the tendency of the price to move in the direction of the jump in the periods after its occurrence. The proposed trading system, working on a given frequency, will thus enter a trade, immediately after a jump is identified, in the direction of the jump, for the closing price of the period in which the jump occurred. The trading system will then hold the position for a fixed number of periods after the jump occurrence, with the number of these periods being a meta-parameter of the model (different for every asset and every frequency), that is optimized based on the criterion of maximum profitability in the in-sample period. Apart from the horizon of the trade, the trading system contains two additional meta-parameters, the number of periods k, used for local volatility calculation in the L-Estimator, and the confidence level p used for jump identification. Both of these parameters will also be optimized in order to maximize the in-sample profitability. The optimization is performed by using a simple grid search, in which different combinations of system parameters are tested and the most profitable one is then used for the out-sample trading. The tested values of the parameters are $k=\{4,8,16,32,64,128,256\}$, $p=\{0.9,0.95,0.99,0.999\}$ and $h=\{1,2,4,8,16\}$, with the horizon h denoting the number of periods at the given frequency that is traded by the system.

The trading system is applied to 7 assets, specifically 4 currency exchange rates (EUR/USD, GBP/USD, USD/CHF and USD/JPY) and 3 futures contracts (Light Crude Oil, E-Mini S&P 500 and VIX Futures), each of them observed at 7 different frequencies (1-Minute, 5-Minutes, 15-Minutes, 30-Minutes, 1-Hour, 4-Hours and 1-Day). Detailed description of the dataset and of the methods used for its processing was already provided in the previous section. For the purposes of the trading system evaluation, the dataset for each of the time series had to further be divided into an in-sample period (first 50% of the data, used for meta-parameter optimization) and out-sample period (last 50% of the data, used for profitability evaluation). All of the profits were calculated in the presence of transaction costs (spread + commission) equal to 1 pip for the currency time series (equal to 10 USD for the EUR/USD exchange rate) and 1 tick + broker commission for the futures contracts (commission was set equal to 2.2 USD per trade and the spreads are assumed to be one tick wide, corresponding to 10 USD for Light Crude Oil, 12.5 USD for the E-Mini S&P 500 contract and 50 USD for the VIX Futures).

The main criterion to evaluate the profitability of the applied trading systems with respect to their risk, will be the Drawdown ratio, defined as the ratio between the profit of the system per annum and its maximum historical drawdown (i.e. the maximum decrease of equity over the whole in-sample or out-sample period). As the maximum drawdown is often used by speculators to determine the minimum amount of capital needed to trade the given system (determining thus the maximum leverage that can reasonably be used by the trader), it is possible to interpret the value of the drawdown ratio as the expected annual return of the system, in the case when the drawdown ratio rule is used to calculate the utilized leverage. As this rule may be rather aggressive for most of the more risk-averse investors, a conservative estimate of the expected return of the system would be one half of the drawdown ratio, corresponding to the invested capital covering double of the maximum historical drawdown ratio.

Table 3 shows the results for all of the trading systems (with optimized meta-parameters on the in-sample period) for all of the currency exchange rates and return frequencies that were tested.

We can see from Table 3 that the proposed system achieved promising results for some of the currencies and frequencies, with the out-sample drawdown ratios ranging from 30% to 50% and achieving even 137% in one of the cases (30-Minute frequency for the CHF/USD rate).

An interesting result is, that at the highest frequency of 1-Minute, the systems achieved significant losses even in the in-sample period. This can be attributed to the effect of the transaction costs, as there is very high number of small jumps identified at these frequency, which do not seem to posses high enough predictive power to cover the transaction costs associated with so many trades. All of the systems achieved thus a loss at the 1-minute frequency.

Table 3 – Profitability of the after-jump trading system applied to the 4 currency exchange rate markets

analyzed series		optimized parameters			in-sample results					out-sample results				
currency	freuency	k_opt	p_opt	h_opt	periods	trades	profit	max DD	DD Ratio	periods	trades	profit	max DD	DD Ratio
EUR/USD	1-Day	16	0,95	16	2035	5	5060	-2465	0,2542	2036	7	10999	-3344	0,4071
	4-Hour	16	0,95	16	12190	127	17360	-5245	0,4099	12190	93	5274	-9795	0,0666
	1-Hour	8	0,99	16	48270	741	44690	-12015	0,4606	48270	802	-15801	-30217	-0,0647
	30-Minute	32	0,95	16	96533	664	29230	-8815	0,4106	96533	682	30999	-11741	0,3268
	15-Minute	64	0,9	16	193058	888	16810	-9045	0,2301	193058	1022	34545	-9916	0,4312
	5-Minute	128	0,999	16	578795	876	4820	-12275	0,0486	578796	1229	12856	-14457	0,1101
	1-Minute	64	0,999	1	2885339	2072	-42560	-42570	-0,1238	2885339	3503	-50517	-51192	-0,1221
GBP/USD	1-Day	4	0,95	16	2035	118	69930	-17260	0,5017	2036	104	-22152	-38795	-0,0707
	4-Hour	4	0,9	16	12190	971	97310	-38385	0,3139	12190	917	76198	-28035	0,3364
	1-Hour	16	0,95	16	48269	485	22010	-9425	0,2892	48269	610	-10680	-29938	-0,0442
	30-Minute	32	0,9	8	96532	596	29490	-8105	0,4506	96532	743	13197	-11687	0,1398
	15-Minute	64	0,9	8	193052	706	16030	-5915	0,3356	193053	943	15763	-11072	0,1762
	5-Minute	128	0,999	16	578778	667	15580	-5655	0,3412	578778	1163	-2457	-15686	-0,0194
	1-Minute	64	0,999	1	2885203	1946	-57500	-57500	-0,1238	2885203	3719	-61410	-61410	-0,1238
USD/CHF	1-Day	8	0,99	8	2035	16	13720	-4005	0,4242	2035	13	-17504	-20059	-0,1080
	4-Hour	4	0,9	4	12188	974	41180	-22410	0,2276	12189	947	-21858	-31677	-0,0854
	1-Hour	4	0,9	16	48260	3468	41430	-48525	0,1057	48260	3245	-30971	-62916	-0,0609
	30-Minute	32	0,95	8	96513	678	43400	-9985	0,5382	96514	722	57277	-5151	1,3763
	15-Minute	64	0,9	16	193007	909	37620	-7705	0,6046	193008	992	37569	-8161	0,5698
	5-Minute	248	0,999	16	578693	917	5200	-15645	0,0412	578694	1132	32507	-8356	0,4815
	1-Minute	64	0,999	1	2889566	2148	-66480	-66480	-0,1238	2889567	3855	-47529	-48033	-0,1225
USD/JPY	1-Day	64	0,99	16	2035	6	11320	-490	2,8608	2036	8	-3618	-14008	-0,0320
	4-Hour	4	0,99	4	12190	762	25350	-7930	0,3959	12190	717	8257	-16127	0,0634
	1-Hour	8	0,99	16	48270	638	25190	-8990	0,3470	48270	641	6064	-15664	0,0479
	30-Minute	32	0,9	8	96533	466	10950	-5050	0,2685	96534	552	-247	-13670	-0,0022
	15-Minute	128	0,9	16	193056	574	10870	-7965	0,1690	193056	660	-6326	-20816	-0,0376
	5-Minute	16	0,999	8	578780	1069	-12500	-16815	-0,0921	578780	1638	-13017	-16722	-0,0963
	1-Minute	128	0,999	1	2884654	1923	-40580	-40585	-0,1238	2884655	2945	-47610	-47955	-0,1229

Large number of small jumps, resulting in too many trades and high transaction costs has apparently have an overly negative effect on the returns on the 5-minute frequency too, as only the CHF/USD currency pair achieved significant profits at that frequency, with EUR/USD achieving slightly lower profits, while the other currencies ended with a loss. The results on the 5-minute frequency are in

slight disagreement with the results in Novotny, Petrov and Urga (2015) who found their after-jump trading system to be profitable on this frequency for multiple currencies. Nevertheless, while the authors work only with a short period of 4-month during the year 2013, while out study is applied to more than a decade of returns, the results are not very comparable.

The two most promising frequencies with regards to the system performance are the 15-Minute and the 30-Minute frequency, at which the tested systems achieved high out-sample profits for all of the tested currencies with exception of YEN/USD. It seems that at these frequencies the jumps are not already so frequent to plague the system with unacceptably high transaction costs, caused by small jumps with low predictive power, but at the same time there is still enough jumps for the system to achieve good profits. At the same time the market can be assumed to not be as efficient on the higher frequencies as on the lower ones, which could further explain why the systems achieved high profits at these two frequencies, while at the lower ones they ended with a loss.

The highest annualized drawdown ratios at the 15-minute and the 30-minute frequencies were achieved by the CHF/USD exchange rate (57% and 137%), followed by the EUR/USD exchange rate (43% and 33%) and then the GBP/USD exchange rate (18% and 14%). For USD/JPY a loss was achieved (-3% and -0.2%).

At the lower frequencies (1-hour, 4-hour and 1-day) the results start to significantly differ for the different currencies and it is thus rather difficult to interpret them with regards to market efficiency (although on average, the profitability seems to decrease, corresponding to more efficient markets at these frequencies). Surprisingly, the 1-hour frequency was unprofitable for all of the currencies with the exception of the YEN/USD for which it achieved a low profit (5% drawdown ratio). At the 4-hour frequency, however, the GBP/USD achieved a high profit (34%), while EUR/USD and YEN/USD achieved low profits (7% and 6%). At the daily frequency, only the EUR/USD exchange rate achieved a profit (40%), which, although being relatively high, was achieved by only 7 trades and it has thus not very significant from the statistical point of view.

Overall, USD/CHF seems to be the most promising currency for our after-jump trading strategy, followed by EUR/USD, then GBP/USD and finally YEN/USD which was the least profitable (this could potentially be explained by the regular monetary interventions of the Bank of Japan into the Yen currency exchange rate, which may results in a different after-jump behaviour of the exchange rate than the standard news-induced jumps). Apart from the outlier-like 137% drawdown ratio for the USD/CHF 30-minute frequency, the trading systems achieved annualized drawdown ratios ranging from 30-50% for multiple other frequencies (primarily for USD/CHF and EUR/USD). This would correspond to 30-50% return per annum in the case of the aggressive use of leverage (capital covering exactly the maximum drawdown), respectively to 15-25% return per annum in the case of the conservative use of leverage (capital covering double of the maximum drawdown). The profitability is thus relatively high and may be interesting to practitioners, especially considering that the simple system that we propose may be further improved.

Table 4 shows the results for the analysed futures contracts.

It is apparent that the results for the futures do not look as good as for the currency exchange rates. The only asset for which the proposed after-jump trading system achieved interesting profits was the Light Crude Oil. Unlike the currency exchange rates, however, the system achieved profit on the 3 lowest frequencies, while at the 15-Minute and 30-Minute frequencies it achieved a loss. The highest absolute profit in the out-sample period was achieved at the 1-hour frequency, it was, however, associated with a massive drawdown, causing the drawdown ratio and thus the risk-adjusted relative

profitability, to not be so good. Best relative, risk-adjusted profitability was achieved on the 1-Day frequency, the profit was, however, caused by only 4 trades, making the result insignificant.

For the E-Mini S&P 500 Futures and VIX Futures the after-jump trading strategy seems to be completely unprofitable on all of the frequencies. The reason for this could be the existence of a significant risk premiums on these markets, which are known to rise when the market crashes (or the VIX index rises). The rise of the risk premium does in these cases increase the negative jump movements (i.e. the market crashes and VIX surges), while at the same time, increasing the expected future returns (i.e. corresponding to a mean-reversing after-jump behaviour of the price). Indeed, a contrarian strategy of buying the stock index after crashes (or shorting the VIX after its spikes) is well known among the hedge funds and individual speculators. It is thus possible to hypothesize that this risk premium mean-reversion effect negated the momentum-like effect of the jumps (which are mostly negative for the S&P 500 and positive for the VIX), causing the proposed after-jump trading system to be unprofitable.

Table 4 – Profitability of the after-jump trading system applied to the 3 futures markets

analyzed series		optimized parameters			in-sample results					out-sample results				
currency	**freuency**	**k_opt**	**p_opt**	**h_opt**	**periods**	**trades**	**profit**	**max DD**	**DD Ratio**	**periods**	**trades**	**profit**	**max DD**	**DD Ratio**
Light Crude Oil	1-Day	256	0,95	16	1273	12	95347,2	-14470,4	0,8159	1274	4	14172,4	-1644,4	1,0667
	4-Hour	8	0,999	16	8864	93	13540,8	-31109,6	0,0539	8865	85	22386	-12851,2	0,2156
	1-Hour	4	0,99	16	37009	2492	64875,2	-65333,2	0,1230	37009	2010	80036	-47690,4	0,2077
	30-Minute	8	0,9	16	73168	1796	-282,4	-62148,4	-0,0006	73169	1107	-47800,8	-53958	-0,1096
	15-Minute	4	0,999	16	144116	10869	10776,4	-110105	0,0121	144116	9068	-265664	-279696	-0,1176
	5-Minute	256	0,999	4	433859	1624	-30015,6	-31029,6	-0,1198	433859	754	292,4	-11990,4	0,0030
	1-Minute	64	0,999	8	2171914	4085	-83844	-85633,2	-0,1212	2171915	2481	-24406,4	-25246,8	-0,1197
E-Mini S&P 500	1-Day	4	0,999	16	2260	83	22522,3	-15103,9	0,1847	2261	109	-67017,1	-68075,2	-0,1218
	4-Hour	16	0,9	16	9438	81	6543,6	-8172,7	0,0991	9438	67	1730,2	-11557,9	0,0185
	1-Hour	128	0,999	4	42424	62	3577,2	-2439,95	0,1816	42425	74	-9413,1	-9850,25	-0,1183
	30-Minute	16	0,9	16	83488	915	14049	-18978,1	0,0917	83489	774	-1668,1	-14174,1	-0,0146
	15-Minute	256	0,999	8	168359	235	991	-8262,95	0,0149	168359	294	-18068,6	-18971,5	-0,1179
	5-Minute	256	0,999	8	504285	641	-9282,9	-10418	-0,1103	504285	755	-15347	-17817,7	-0,1066
	1-Minute	128	0,999	1	2522538	2258	-44572,7	-45098,9	-0,1224	2522539	2069	-49028,6	-49665,1	-0,1222
VIX	1-Day	8	0,99	4	791	10	22676	-27,2	103,2366	791	10	-2988,4	-4702,4	-0,0787
	4-Hour	4	0,9	4	1352	114	22168,4	-8712	0,3151	1353	100	-6780	-9835,6	-0,0853
	1-Hour	8	0,9	16	5473	137	4957,2	-15501,2	0,0396	5473	119	-18373,6	-20819,2	-0,1092
	30-Minute	32	0,99	4	10775	77	3641,2	-8494,8	0,0531	10775	37	-2812,8	-3398	-0,1025
	15-Minute	16	0,99	16	22171	213	9652,8	-10740	0,1113	22171	141	-17610,4	-18794,4	-0,1160
	5-Minute	128	0,999	16	65713	234	19270,4	-7349,2	0,3247	65713	68	-5119,2	-6670,4	-0,0950
	1-Minute	256	0,999	16	329149	683	-76985,2	-77259,2	-0,1234	329150	174	-16505,6	-16505,6	-0,1238

Conclusion

A profitability of a trading system based on the momentum-like effect of price jumps was tested on the time series of 7 different assets (EUR/USD, GBP/USD, USD/CHF and USD/JPY exchange rates and Light Crude Oil, E-Mini S&P 500 and VIX Futures), in each case for 7 different frequencies (1-Minute, 5-Minute, 15-Minute, 30-Minute, 1-Hour, 4-Hour and 1-Day), ranging in the period between 1.11.1999 and 12.6.2015 for the currencies, 2.1.1987 to 4.12.2015 for Light Crude Oil Futures, 11.9.1997 to 4.12.2015 for E-Mini S&P 500 Futures and 26.3.2004 to 2.12.2015 for VIX Futures.

The trading system proceeded by entering long or short trades in the direction of price jumps, for the closing price of the period in which the jump occurred. The position was held for a fixed number of periods (1 to 16), with the number optimized to maximize the in-sample profitability of the system for each of the currencies. The jumps in the time series were identified with the nonparametric and model-free *L*-Estimator, whose parameters (period *k* for local volatility calculation and confidence level *p* for jump identification) were optimized on the in-sample period too. The evaluation of the results was performed on the out-sample period (last 50% of the data for each of the time series), based on the total profitability and the drawdown ratios (yearly profit to maximum historical drawdown). The testing included realistic transaction costs (commissions and spreads), equal to 1 pip for each of the currencies and 1 tick spread + commission (2.2 USD per trade) for the futures.

The results of the study show that the after-jump trading according to the proposed system represents a promising trading strategy for 3 out of the 4 analysed currency markets, with the highest profitability achieved on the USD/CHF series, followed by EUR/USD and then GBP/USD, while for USD/JPY rate achieved mostly loses. The most profitable frequencies for trading were for most of the currencies the 15-Minute and the 30-Minute Frequency at which the system achieved (for USD/CHF and EUR/USD) returns in the range of 30-50% in the case of heavy use of leverage (equity covering exactly the maximum drawdown) or 15-25% in the case of lower use of leverage (equity covering the double of the maximum drawdown). Similarly high profits were also achieved on the USD/CHF series on the 5-minute frequency and on the GBP/USD series on the 4-hour frequency. The 1-minute frequency proved to be unprofitable due to too many small jumps with low predictive power, plaguing the system with large transaction costs. On the 1-Day frequency there was generally too few jumps in the time series to view the results as significant, but the EUR/USD and Light Crude Oil achieved relatively high relative profits even at this frequency.

As for the Futures markets, the only one at which the system was profitable was the Light Crude Oil at frequencies of 1-Hour or larger. At 1-Hour and 4-Hour frequency the system achieved 20% yearly returns (in the case of high leverage), respectively 10% (in the case of lower leverage). The relative profitability on the 1-Day frequency was even higher but insignificant due to small number of trades.

The system achieved mostly losses for the E-Mini S&P 500 and VIX futures markets, which we attribute to the rising risk premium in the case of market crashes (negative jumps for the stock index and positive jumps for the VIX), which is making the contrarian approach (i.e. buying the index in the case of negative jumps, respectively shorting the VIX in the case of positive jumps) a potentially more suitable strategy for these markets than the momentum-like trading that proposed in this study.

As for future areas of research, the profitability of the model could most probably be increased by using a more sophisticated exit strategy (i.e. stop-loss and profit-target) as well as by using a more sophisticated optimization method for the model parameters, potentially trading with multiple values of the parameters at once. Similarly, an application to other assets and asset classes would be a straightforward extension of the research which might offer potential new insights and trading opportunities.

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